Continuous tensor networks for QFT renormalization

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Based on joint work with Qi Hu and Guifré Vidal

Introduction

The computational implementation of the **renormalization group flow** is an interesting problem with numerous applications: study of phase diagrams, optimization of algorithms for numerical simulation of many-body systems, holography...

In the past decades, algorithms inspired on **tensor networks** have made it possible to realize nonperturbative RG flows for systems defined on a **lattice**.

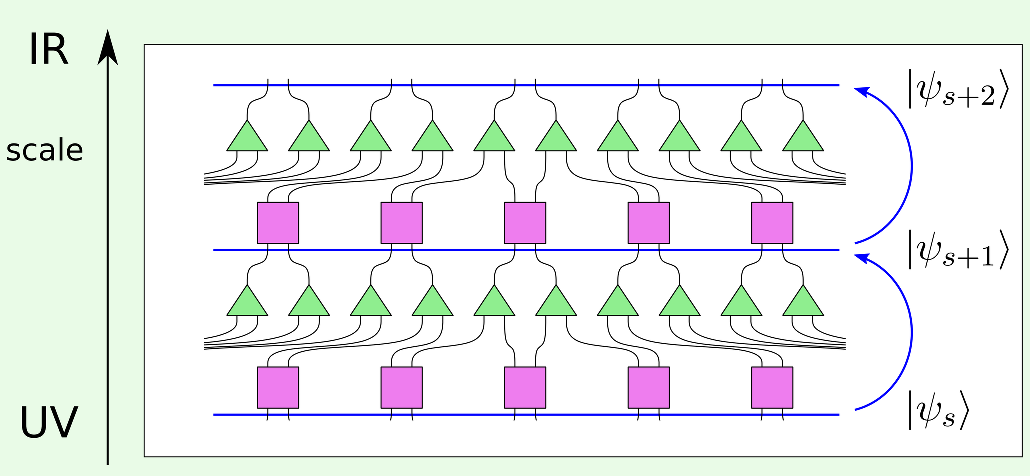
More recently, the research program of **continuous tensor networks** aims to translate these techniques to the language and setting of **QFT**, allowing in particular to implement them while fully respecting the translational and rotational symmetries of the theory.

Lattice | Continuum

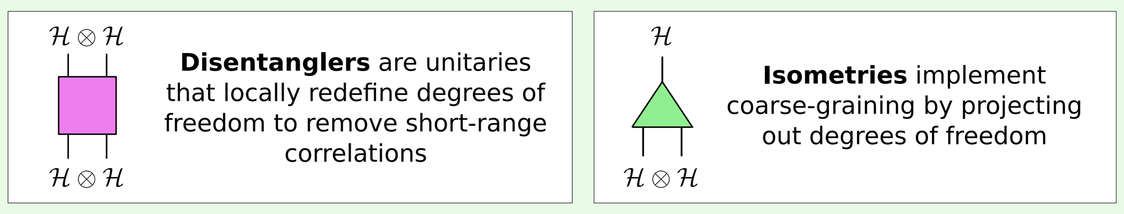
Quantum states

Multiscale Entanglement Renormalization Ansatz (MERA [1])

The MERA is a tensor network that describes an RG flow of a quantum state on the lattice. It is given by a **sequence of linear maps**, each representing a coarse-graining step:



These maps are made of compositions of disentanglers and isometries:

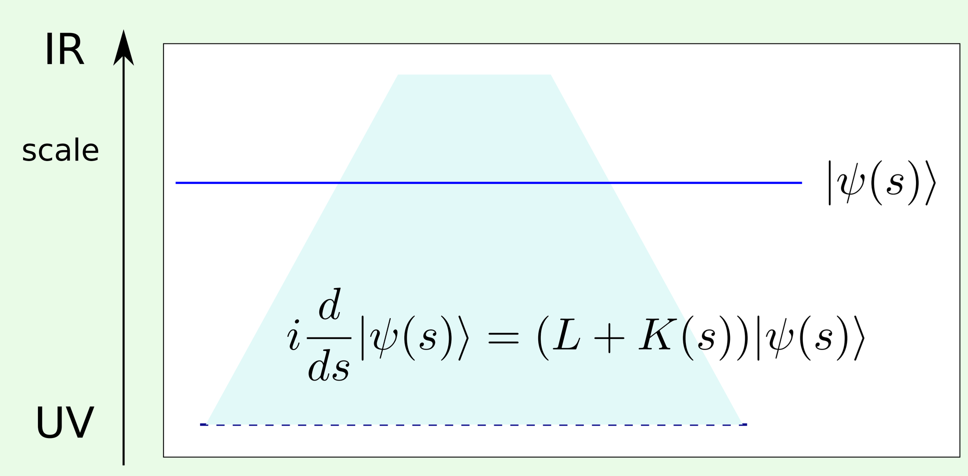


→ Disentanglers and isometries are determined **variationally** by the algorithm

MERA has been successful at recovering the **conformal data** that describe quantum critical points (RG fixed points), thus providing evidence of its validity as an implementation of RG flow.

continuous MERA (cMERA [2])

The cMERA is a construction inspired in MERA that describes an RG flow for a quantum field theory state*. It is implemented by a **unitary evolution in scale**:

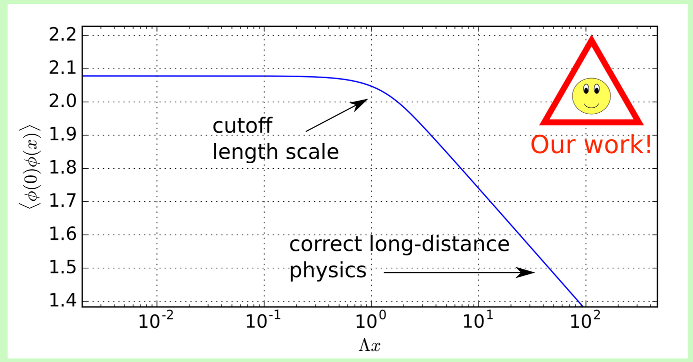


The generator contains two contributions:

L is the generator of spacetime rescalings

$K(s)$ is a quasi-locally supported operator which should be determined variationally and performs the required removal of short-range correlations

* cMERA states are special: they are endowed with an intrinsic **UV cutoff** (analogous to a lattice spacing). For example, their correlators become trivial below a certain length scale [3]:



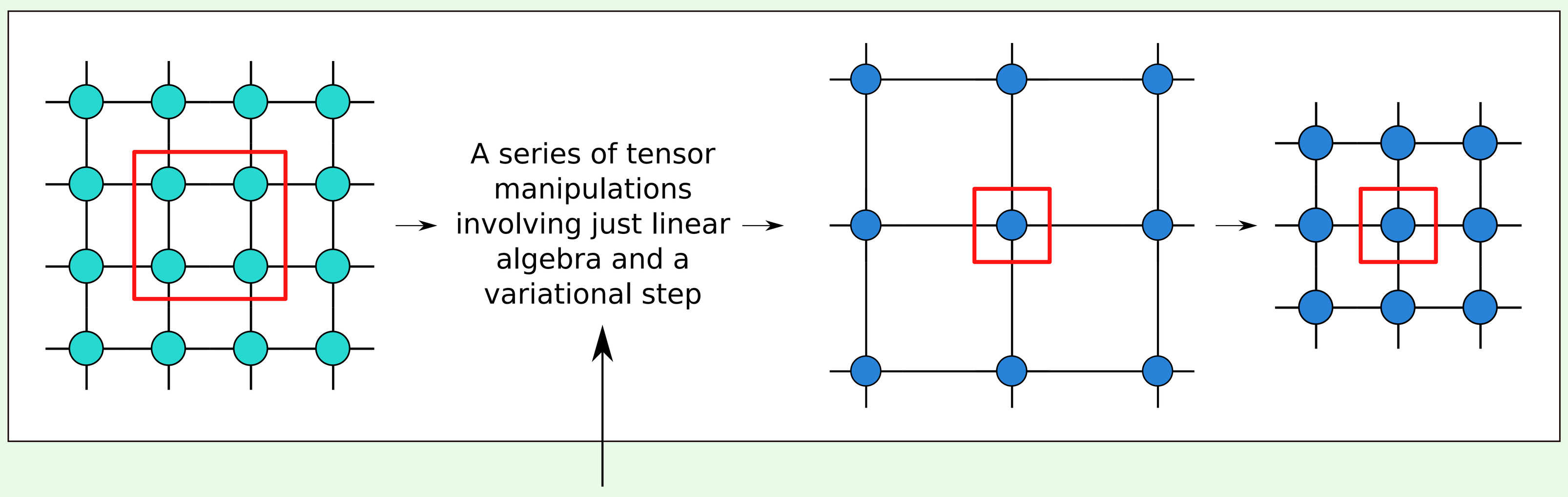
The cutoff is **invariant** under the RG flow.

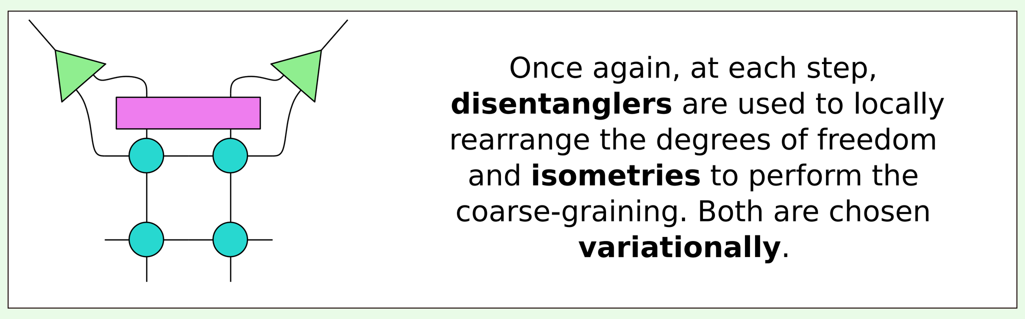
While an entirely variational approach is still not available, cMERA is currently understood in the particular case of **free** theories, for which the conformal structure of the RG fixed points has been shown to be retrievable from the cMERA [4].

Partition functions / Euclidean path integrals

Tensor Network Renormalization (TNR [5])

TNR is an algorithm that implements an RG flow on classical partition functions / Euclidean path integrals written as a tensor network resulting from the repeated contraction of a single tensor along the directions of a lattice. The result is an **evolution in the space of tensors**:





TNR builds on former proposals ([6]) and goes beyond them: it manages to retrieve the **correct structure of RG fixed points**, thus providing a grasp on the phase diagram of lattice systems.

continuous TNR (cTNR [7])

Our work!

cTNR is a construction that implements an RG flow on classical partition functions / Euclidean path integrals resulting from integrating the exponential of an action* over the spacetime manifold. The result is an **evolution in the space of actions**:

$$Z_0 = \int [d\phi]\, e^{-S_0[\phi]} \longrightarrow Z_s = \int [d\phi]\, e^{-S_s[\phi]}$$

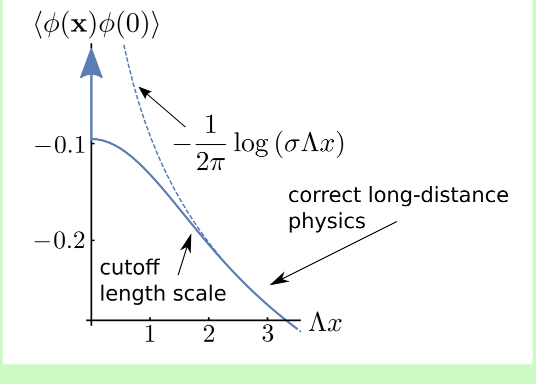
$$\frac{d}{ds} S_s[\phi] = (L + K(s)) S_s[\phi]$$

The generator contains two contributions:

L is the generator of spacetime rescalings

$K(s)$ is a quasi-locally supported operator which should be determined variationally and performs the required local rearrangement of degrees of freedom

* cTNR actions are also special: they also present a UV cutoff due to them being **quasilocal**, i.e. smeared nonlocally within a certain length scale, which also shows up in the correlators:



The cutoff is **invariant** under the RG flow.

As for cMERA, a general variational version of cTNR is yet to be understood outside free theories. However, **cTNR matches cMERA's success** at reproducing the conformal structure of free RG fixed points, thus opening the door to future developments on the nonperturbative implementation of RG flow directly in the continuum.

References

[1] G. Vidal, *A class of quantum many-body states that can be efficiently simulated*, Phys. Rev. Lett. 101, 110501 (2008)
[2] J. Haegeman, T. J. Osborne, H. Verschelde, F. Verstraete, *Entanglement renormalization for quantum fields*, Phys. Rev. Lett., 110, 100402(2013)
[3] **A. F.-R.**, G. Vidal, *Entanglement structure and UV regularization in the continuous multi-scale entanglement renormalization ansatz*, J. High Energ. Phys. (2017) 12, 129
[4] Q. Hu, G. Vidal, *Spacetime symmetries and conformal data in the continuous multi-scale entanglement renormalization ansatz*, Phys. Rev. Lett. 119, 010603 (2017)
[5] G. Evenbly, G. Vidal, *Tensor network renormalization*, Phys. Rev. Lett. 115 (18), 180405 (2015)
[6] M. Levin, C. P. Nave, *Tensor renormalization group approach to 2D classical lattice models*, Phys. Rev. Lett. 99, 120601 (2007)
[7] Q. Hu, **A. F.-R.**, G. Vidal, *Continuous tensor network renormalization for quantum fields*, arXiv:1809.05176

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